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A Note on the Paper by K. Shulev "A Solution of the Stokes' Problem for a Circular Cylinder"

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In the said paper [1] the ambitious task is attemped to prove that the well-known paradox of Stokes' does not actually exist. This paradox is founded in the fact that a solution to the Stokes equations with proper behaviour at infinity can not be found when the two-dimensional flow around circular cylinder is considered [2]. Naturally, in the course of thinking the sought solution for the stream function is thought of to possess fourth-order derivatives. As usual, due to the uniqueness of the generalized solution [3] it turns out that the smooth solution coinsides with the generalized one and hence the former is the unique solution of the Stokes problem and no other (even discontinuous) solutions are not to be expected.

It is well known that for the Stokes flow around circular cylinder a smooth solution satisfying the conditions at the body does exist (see [4]), but its asymptotic behaviour in the far field $r \to \infty$ is $r \ln r$, i. e. the flow there is not the indisturbed uniform flow. In the light of what is said above one is to expect that this is the only solution to the specific Stokes' problem under consideration.

Still in [1] is attemted obtaining a very special kind of solution exhibiting singularities at two specific lines $\theta=0$ and $\theta=\pi$. Out of a sudden the function $\sin\theta$ is developed into two different series each of them valid only in the respective subinterval $(0, \pi)$ and $(-\pi, 0)$ (see eqs (15), (16) of [1]). The resulted approximation for $\sin\theta$ obviously lacks the first derivatives at $\theta=0$ and $\theta=\pi$. In apparent attempt to smooth this approximation a four-fold integration of the series for $\sin\theta$ is invoked in [1] and the representations (17), (20) are derived.

As it should have to be expected, however, the solution obtained in [1] inherits the above discussed singularities despite all these measures. And the amazing thing is that this is acknowledged in the very paper [1], where the velocity component v_θ is shown to be discontinuous for $\theta=0, \pi$ and is defined there through arbitrary considerations.

So far, one could be temted to conclude that if not classical, at least a new generalized in a sense solution for the Stokes' flow around circular cylinder is obtained in [1] and hence that a counter-example for the uniqueness of the generalized solution is raised. Unfortunately, even that is not true, because there exist errors in the way the solution is obtained in [1].

The problem is that the set of functions $\{const, \theta, \theta^2, \theta^3, \theta^4\}$ is not independent of the set $\{\cos 2s\theta\}$. An excellent opportunity to show that provides the comparison between eqs (15) and (17) of [1] which gives

$$-\frac{\pi}{6}-\frac{\pi^3}{360}+\theta-\left(\frac{2}{\pi}-\frac{\pi}{6}\right)\frac{\theta^2}{2}-\frac{\theta^3}{6}+\frac{\theta^4}{12\pi}=\sum_{s=1}^{\infty}\frac{(2s)^4-1}{(1-4s^2)(2s)^4}\cos 2s\theta=-\sum_{s=1}^{\infty}\frac{4s^2+1}{16s^4}\cos 2s\theta.$$

The last equality means that at least one of the functions $\{const, \theta, \theta^2, \theta^3, \theta^4\}$ is not independent of the set $\{\cos 2s\theta\}$. If the last assertion holds, the whole development (23) together with the scheme for identification of coefficients (26)-(31) are compromised. That is the cause for the failure to obtain in [1] terms of type $\xi \ln \xi$ corresponding to the mentioned in the above $r \ln r$. Regardless to all equilibristics performed in [1] the said terms are still to be present at least in the open intervals $(0, \pi)$ and $(-\pi, 0)$ if the solution is correctly calculated. The latter stems from the fact that in these open intervals the adopted in [1] series for $\sin\theta$ are convergent and hence the obtained solution must approximate the form $f(r)\sin\theta$ for which is well known [4] that it includes term of type $r \ln r$. This is one more point of view to fine reasons why the generalized function (23) from [1] is not a generalized solution to the Stokes' problem for a circular cylinder.

Thus we arrive to the conclusion that the paper under consideration [1] presents no any proofs that the paradox of Stokes does not exsit.

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