Numerical Solution of 2-D Helmholtz Problem for Blunt Bodies and Airfoils

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Summary

The classical problem of discontinuous ideal flow that consists of potential streaming and stagnation zone (Helmholtz problem) is treated numerically. In terms of deformed parabolic coordinates a difference scheme for solving Laplace equation in between the unknown separation boundaries is devised. A simple iterative procedure is employed for defining the separation point. The presented algorithm predicts for case of circular cylinder an angle of separation as high as 82° (measured from the leading stagnation point) in contrast to the quantity 55° resulting from the approximate implementation of hodograph method. Results obtained for the drag of circular cylinder are in very good quantitative agreement with the experiment while the solution with angle of separation equals to 55° predicts only the half of the quantity. The capability of the numerical technique proposed is displayed also for the case of NACA 2418 airfoils where the drag is predicted with 90-95% accuracy and the lifting force - with 63%.

Introduction. The notion of discontinuous ideal flow was introduced by Helmholtz [1] in order to explain the fact that even for vanishing viscosity the moving liquid exerts a sizable force on submerged bodies. In other words, for very high Reynolds numbers it is not the potential flow but the Helmholtz flow which is expected to be the more adequate "outer solution" of asymptotic expansion to which the "inner solution" (boundary layer) should match. For bodies with sharp eddies when the position of separation point is obvious the idea of discontinuous ideal flow was immediately applied with great success by Kirchhoff [2]. It was not the case, however, with the smooth blunt bodies for which the separation point can be obtained only after imposing certain additional conditions; the most natural of the latter is the so-called condition of smooth separation introduced by Brillouin and

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Villat (see [3]). This condition was implemented for the flow around a circular cylinder by Brodetsky [4] but the result for the drag coefficient did not live up to the expectations remaining two times smaller than the experimental quantity. The magnitude of the drag coefficient is defined solely by the position of separation point. The Brodetsky's result was $55°$ (measured from the leading stagnation point). The recent high-quality numerical calculations of Navier-Stokes equations for Reynolds numbers as high as 600 conducted by Fornberg [5] indicate that the separation point may be as far as $95°$ from the leading stagnation point. These facts apparently suffice to dismiss the discontinuous Helmholtz flow as a model capable of quantitative predictions for high Reynolds number flows. The main idea of the present work is to use another approach to numerical solution of Helmholtz problem because the truncated-power-series approximation in [4] and in works following the same technique (see, e.g. [7]) is rough while the position of the separation point may prove extremely sensitive.

Method of solution Guided by the above considerations that the troubles are founded in the inverse nature of the hodograph method we decided to attack the problem directly. For this reason in [8,9,10] is developed finite difference scheme for solving Laplace equation in terms of deformed parabolic coordinates which are topologically most suited for infinite stagnation zones. The coordinates are scaled by the shape function of the unknown boundary of the separation zone which is essential for having a computational domain with fixed boundaries. By means of the above difference scheme the Helmholtz problem can be solved for a given magnitude of separation angle θ. For those values of θ for which the separation is not smooth the solution is artificially smoothed because of the approximate properties of the scheme. Special iteration procedure is devised for calculating θ (see [8,9,10]).

Results, comparisons, discussion By means of the cited numerical technique we obtained two different solutions for the unknown boundary which did not surprise being reminded that the latter is calculated from the quadratic Bernoulli integral. Concerning the solution with expanding stagnation zone (called solution II throughout the work) the separation angle is $75°$ which is much closer to the eventual asymptotic value for high *Re* of Fornberg's [6] results (see Fig. 1). The respective result for the drag coefficient is really delightful (Fig. 2) which allows us to interpret solution II as the limiting ideal flow for the case of so-called laminar separation. Respectively the solution with contracting stagnation zone (solution I) is interpreted as the limiting ideal flow when turbulent separation takes place. This notion is supported by the fact (Fig.2) that the predicted by solution I drag coefficient compares quantitatively very well with the experimental value in the region of the so-called crisis of resistance.

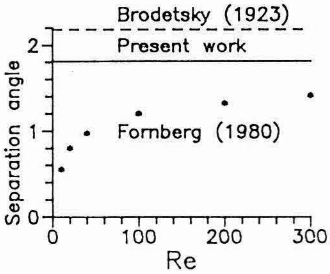

Fig.1

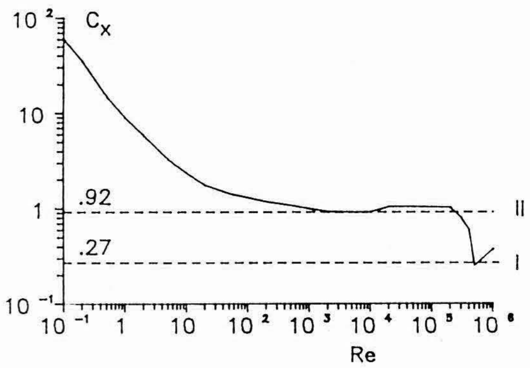

Fig.2

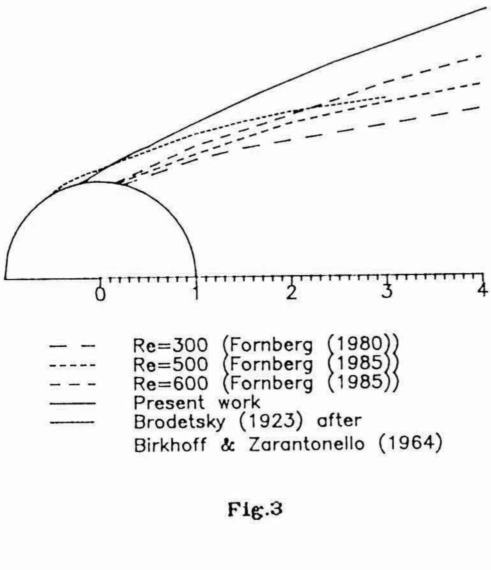

Fig.3

In Fig. 3 and Fig. 4 the shape of stagnation zone and the pressure distribution are compared with viscous simulations and earlier results [4], respectively. It is seen that the agreement is satisfactory. Fig. 5 has the special purpose to show that the obtained here $\theta=75°$ gives a really smooth separation while the pressure distribution calculated with $\theta=55°$ obviously lacks smoothness at the separation point. In other words the condition of Brillouin-Villat is in fact not satisfied for $\theta=55°$ although this value was obtained in apparent attempt to satisfy exactly that condition [4].

One of the main advantages of the proposed numerical approach is that unlike the hodograph method it can be extended to bodies of arbitrary cross sections without principal changes in the particular implementation. In authors work [11] a number of specific airfoils are treated and promising results are obtained for drag coefficient, but the lifting force was well bellow the experimental value. After some improvements of the coordinate system are introduced (above mentioned deformed parabolic coordinates) the lifting force becomes about 63% of the measured value [12](see Fig. 6). The details on new coordinate system are to be published in a separate paper elsewhere. Here is to be only mentioned that special care is taken to adequately represent the regions with high gradients (see Fig. 7) by means of non-uniform meshes (see [11]) and other numerical means.

Conclusion The good quantitative or qualitative agreement with known experimental data for the drag coefficient or lifting force for such complicated geometry as the airfoil allows us to claim that the Helmholtz model is much more adequate that it has been thought out till now. We are confident that the limitations of the numerical technique of finite differences in the harsh geometrical circumstances are those, which introduce the main error because in the case of simple geometry - circular cylinder - the agreement for the force is excellent. Being reminded that the computational time is an order of magnitude smaller than the respective value for viscous simulations the practical perspectives of reviving the Helmholtz model as tool for express and accurate enough assessment of properties of flow around blunt bodies are obvious.

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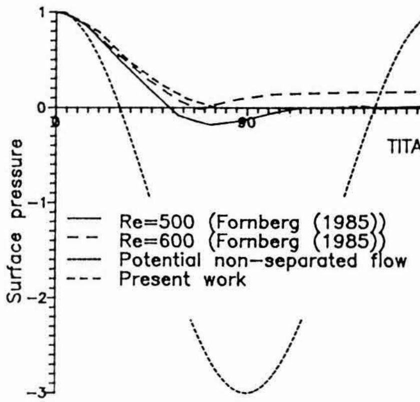

Fig.4

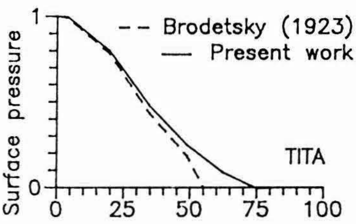

Fig.5

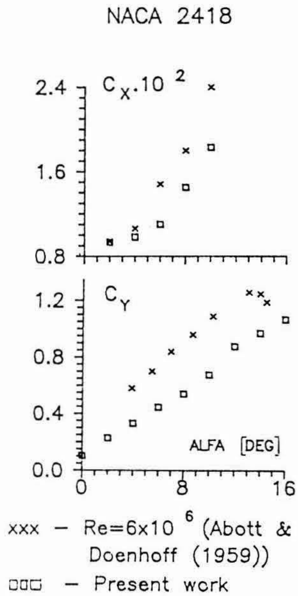

Fig.6

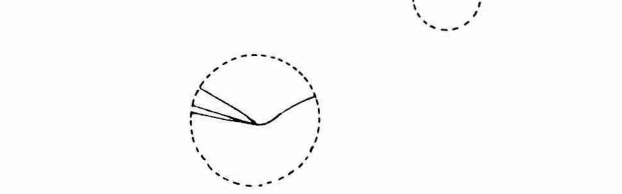

Fig.7

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