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PREFACE

The Eurotherm Committee was created in Brussels in October 1986, following an initiative from members of the European Community, taken at the 8th International Heat Transfer Conference in San Francisco earlier that year. The aim of Eurotherm is to promote and foster the European cooperation in Thermal Sciences and Heat Transfer by gathering together scientists and engineers working in specialized areas, through scientific events such as seminars and wider conferences.

Since that time, more than fourty seminars were organized, dealing with many different topics related to heat transfer. A former seminar, n° 13, was devoted to particular aspects of modelling in Computational Fluid dynamics (Harwell-UK, 1990). The present seminar n° 36 held in ENSMA-Futuroscope (September 1994) dealt with advanced concepts and techniques in thermal modelling.

The objectives of the seminar were to present new numerical approaches or conceptual tools that either reduce the computational time (with the control of a possible loss of accuracy) or allow a more accurate description of thermal systems.

Three complementary approaches were explored :

- *at system level (bondgraphs, network representations, model reduction, expert systems)*
- *at the classical macroscopic level based on the continuum mechanics description (combining control volume and finite element methods, multigrids)*
- *at a microscopic level, where some new and promising techniques were identified, like molecular dynamics or cellular automata.*

The four invited lectures presented in these proceedings did contribute to review those new fields of interest for heat transfer modelling.

Besides, many other valuable papers have been collected here, as advanced contributions in :

- *numerical techniques (11 papers)*
- *system and processes analysis (8 papers)*
- *model reduction and reduced schematisation (6 papers)*
- *data processing (3 papers)*
- *improved modelling on a physical basis (3 papers)*
- *thermal radiation modelling (5 papers)*

This Seminar 36 was actually an international event, attended by 90 scientists and engineers from 10 countries, not only from the European Community. Four invited lectures were delivered, 25 papers were presented in oral sessions and 31 in poster sessions. Almost all the authors proposed a full length paper, among which 40 have been accepted after the final review, processed by the Scientific Committee, with the help of some external relevant personalities from the areas.

Denis Lemonnier, Jean-Bernard Saulnier and Martin Fiebig, editors

Splitting methods for free-surface viscous flows subjected to thermal Marangoni effect

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Patterned convection driven by surface-tension gradients in a thin layer of a Boussinesq fluid open to air is considered. The evolution of the cellular structure is investigated numerically using the $(1+2)D$ equation derived by Knobloch and Shtilman-Sivashinsky (for infinite Prandtl number fluids). For the numerical solution of the equation considered, a splitting difference scheme is developed. The computational domain is a large box. Two distinct cases are considered: (a) the material of lower boundary is much better heat conductor relative the fluid; (b) the other extreme case, a poorly conducting lower wall. For the first case our calculations confirmed the existence of a steady hexagonal-pattern as observed in carefully controled laboratory experiments. In the second case a complex time-dependent pattern of irregular polygons takes place, which was also reported in experimental works for similar conditions.

1 Introduction

Pattern formation in driven systems with many degrees of freedom (e.g., continuous systems) is a paradigmatic example of self-organization, the latter describing the emerging properties that result from the interaction of large number of units (degrees of freedom). Rayleigh-Bénard buoyancy-driven convection in horizontal fluid layers (see [7, 12] for theoretical and experimental description) is an instructive particular case.

The main obstacle to theoretical model convetion problems is that the flows are spatially three-dimensional and unsteady $[(1+3)D$ – for brevity]. The natural convective patterns, i.e., those arising from the amplification of initial inhomogeneities are rarely made of a few number of modes. The dynamics of such confined systems is described by a few number of variables, uniform across the system. A popular model is that of Swift and Hohenberg [11] who derived simplified $(1+2)D$ equation for the amplitude of the planform solving for the bulk flow in the framework of the long-wave approximation and infinite Prandtl number. Later on Knobloch [4] and Shtilman and Sivashinsky [10] went on to derive the respective $(1+2)D$ equation for the case of surface endowed with Marangoni effect (Bénard-Marangoni convection).

The price one has to pay when reducing the original $(1+3)D$ problem to $(1+2)D$ simplify equations is increasing the order of spatial derivatives. In Swift-hohenberg (SHE) and Knobloch-Shtilman-Sivashinsky (KSSE) equations, diffusion terms appear with fourth-order spatial derivatives. The latter poses aditional difficulties when devising difference schemes and algorithms for numerical solution. For the SHE we have devised a splitting scheme [2]. In the present work we generalize the scheme to the case of KSSE when the interaction with lower-order but highly nonlinear terms is of importance.

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2 Posing the Problem

The method, already developed by Poincaré and Lindstedt, consists in the perturbative development of the solution near the bifurcation threshold and relating the order of magnitude of the the amplitude of the unstable mode to the distance of the control parameter from the bifurcation point. Experimental evidence shows that the finite amplitude of an unstable mode evolves in space, in a scale much longer than the characteristic wavelength of the mode. Then a complex equation for the amplitude planform can be derived (see [5], and specially [6, 9]) which could be of different type (parabolic or hyperbolic) in different limits.

Systems consisting of a thin layer of a fluid confined between poorly conducting boundaries display a convective structure with a wavelength much longer than the depth of the cell [8]. If in addition the Prandtl number of the fluid is high, then the velocity field rapidly adjusts to any perturbation of the temperature field, which is the *slow* variable, hence dominant of the system. The evolution of the deviation of the temperature field from the base state (conductive state) is described by the $(1+2)D$ equation derived by Knobloch [4]:

$$\frac{\partial u}{\partial t} = -\alpha u - 2\Delta u - \Delta^2 u + \nabla \cdot \left(|\nabla u|^2 \nabla u - \lambda \Delta u - \nabla u - \mu \nabla |\nabla u|^2 \right), \tag{1}$$

where α represents both the distance to the critical Marangoni number and the scaled Biot number. The rest state of the equation (1) is stable for $\alpha > 1$, while the critical wavelength vanishes when $\alpha = 0$. The linear terms contain the selection mechanism of the critical wavelength, and λ and μ represent the asymmetry in the boundary conditions at the top and the bottom. This equation has been numerically solved by Shtilman and Sivashinsky [10] for $1 < \alpha < 0.6$ in a relatively small region (square box), with periodic boundary conditions. These authors obtained a hexagonal pattern for values of α close to 1 and noted a tendency of cells to undergo distortions as α approaches 0.6. Here, following [10] we set $\lambda = \frac{1}{8}\sqrt{7}$, $\mu = \frac{3}{4}\sqrt{7}$.

3 Difference Scheme and Algorithm

To elucidate how the scheme is constructed we rewrite eq. (1) in the following form:

$$\begin{aligned} \frac{\partial u}{\partial t} &= -\Delta^2 u + \nabla \cdot [Q^n(x,y)\nabla u] - \alpha u - \nabla \cdot [F^n(x,y)\nabla u] - \mu \Delta |\nabla u|^2, \qquad (2) \\ F^n &\equiv 2 + \lambda \Delta u^n, \quad Q^n \equiv (\Delta u^n)^2 . \end{aligned}$$

The main idea is to invert on the "new" time stage $n+1$ the negative definite operators in order to avoid the limitations on the time increment of the type $\tau \sim \mathcal{O}(h^4)$ while the non-definite lower-order operators are kept on the "old" time stage n. Then the desired semi-implicit scheme reads:

$$\frac{u^{n+1} - u^n}{\tau} = (L_1 + L_2)\, u^{n+1} + L_{12} u^n - \nabla \cdot [F^n(x,y)\nabla u^n] - \mu \Delta |\nabla u^n|^2, \tag{3}$$

where L_1, L_2 and L_{12} stand for the difference approximations of the operators

$$-\frac{\partial^4}{\partial x^4} + \frac{\partial}{\partial x} Q^n \frac{\partial}{\partial x} - \frac{\alpha}{2}, \quad -\frac{\partial^4}{\partial y^4} + \frac{\partial}{\partial y} Q^n \frac{\partial}{\partial y} - \frac{\alpha}{2}, \quad \text{and } L_{12} = -2\frac{\partial^4}{\partial x^2 \partial y^2},$$

respectively.

The spliting scheme is implemented in a similar manner as in [2], namely

$$\frac{\tilde{u}-u^n}{\tau} = L_1\tilde{u} + (L_2+L_{12})u^n - \nabla\cdot[F^n(x,y)\nabla u^n] - \mu\Delta|\nabla u^n|^2, \qquad (4)$$

$$\frac{u^{n+1}-\tilde{u}}{\tau} = L_2\left(u^{n+1}-u^n\right) \qquad (5)$$

To show that the splitting scheme approximates the original implicit scheme we rewrite (4), (5) as follows

$$\begin{aligned}[E-\tau L_1]\,\tilde{u} &= [E+\tau L_2]\,u^n + \tau L_{12}u^n - \tau\nabla\cdot[F^n(x,y)\nabla u^n] - \tau\mu\Delta|\nabla u^n|^2,\\ [E-\tau L_2]\,u^{n+1} &= \tilde{u} - -\tau L_2 u^n.\end{aligned}$$

Now the intermediate-stage variable $\tilde{u}$ can be eliminated to obtain

$$\left[E+\tau^2 L_1 L_2\right]\frac{u^{n+1}-u^n}{\tau} = (L_1+L_2)\,u^{n+1} + L_{12}u^n - \nabla\cdot[F^n(x,y)\nabla u^n] - \mu\Delta|\nabla u^n|^2, \quad (6)$$

Using $Q_{ij} \equiv (\nabla u^n)^2_{ij}$ one shows that the approximation, preserving on the difference level the differential equation is

$$\frac{\partial}{\partial x}\left[Q^n\frac{\partial}{\partial x}\tilde{u}\right] = \frac{1}{h_x}\left[\frac{Q^n_{i+1,j}+Q^n_{i,j}}{2}\frac{\tilde{u}_{i+1,j}-\tilde{u}_{i,j}}{h_x} - \frac{Q^n_{i,j}+Q^n_{i-1,j}}{2}\frac{\tilde{u}_{i,j}-\tilde{u}_{i-1,j}}{h_x}\right], \quad (7)$$

$$\frac{\partial}{\partial y}\left[Q^n\frac{\partial}{\partial y}u^{n+1}\right] = \frac{1}{h_y}\left[\frac{Q^n_{i,j+1}+Q^n_{i,j}}{2}\frac{u^{n+1}_{i,j+1}-u^{n+1}_{i,j}}{h_y} - \frac{Q^n_{ij}+Q^n_{ij-1}}{2}\frac{u^{n+1}_{i,j}-u^{n+1}_{i,j-1}}{h_y}\right]. \quad (8)$$

4 Results and Discussion

We solved the equation (1) in two large boxes with rigid sidewalls, using the finite-difference method and the semi-implicit intergration scheme above described. The initial condition consisted of a random disturbance with zero mean. Boundary conditions were assumed as $u = \partial u/\partial n = 0$, where n is the direction normal to the the sidewall. These boundary conditions apply to the amplitude of convective patterns [1] and to the Swift-Hohenberg model [3]. The time evolution of the pattern was monitored through a norm defined by

$$||u||_{L_1} = \frac{1}{\tau}\frac{\sum_{i,j}|u^{n+1}_{i,j}-u^n_{i,j}|}{\sum_{i,j}|u^{n+1}|}, \qquad (9)$$

where $u^n_{i,j}$ represents the value of the temperature deviation in the time step n, at point i,j of the horizontal grid. $u^{n+1}_{i,j}$ represents the temperature in the subsequent time step. This norm essentially measures the rate of change of the distancce between two successive matrices containing the temperature values in the points of the grid. It is sensitive to both the growth of the patterns towards a saturated state *and* the phase evolution, which is usually much slower than the saturation.

In the first run we considered the case of good conducting horizontal boundaries, by setting $\alpha = 0.9$. A square box where the length of each side is equal to 150, corresponding to approximately 21 wavelengths, was used. A grid of 402×402 points was defined, leading to a mesh of approximately 20 points per wavelength. In Fig. 1 is shown the time evolution of the system starting from a random initial condition. The system evolves towards a stationary state.

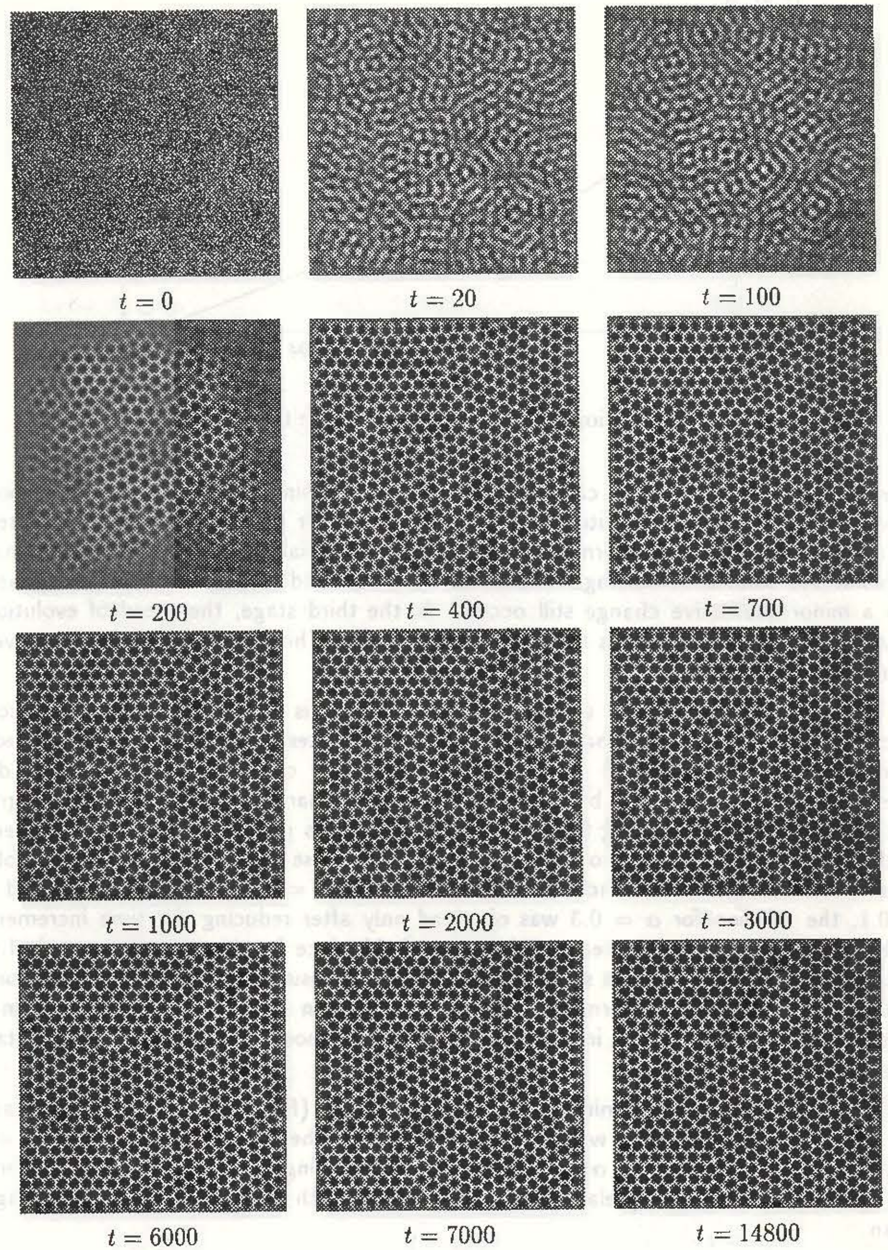


Figure 1: Evolution of the solution of eq. (1) from a random initial condition when the damping is considerable $\alpha = 0.9$.

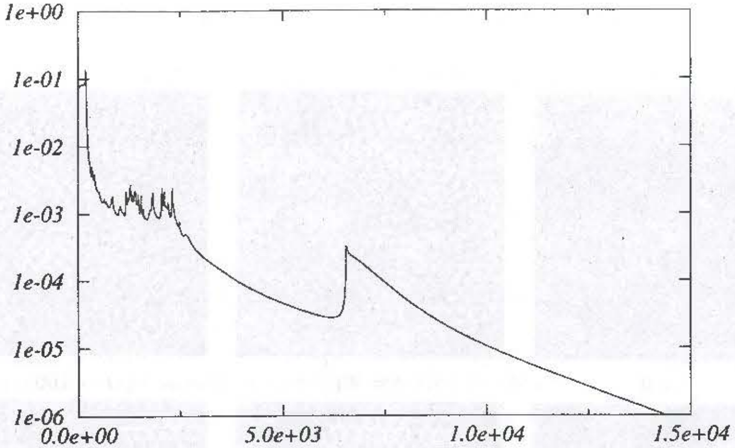


Figure 2: Evolution of the L_1 norm with the time for $\alpha = 0.9$.

Several penta-hepta defects can be observed in the grain boundaries. The evolution of the norm L_1 (Fig. 2) shows initially an irregular behaviour with sharp peaks associated to qualitative changes in the pattern during saturation, essentially consisting of the elimination of most of the defects. This stage is followed by a slow and long evolution of the phase, in which a minor qualitative change still occurs. In the third stage, the speed of evolution of the pattern exponentially decays towards a steady state. This type of pattern is observed in carefully controlled experiments.

It is worth mentioning that unlike the SHE, eq. (1) is non-variational, leading to the possibility of a more compex behaviour in which steady states may not exist. This is precisely the situation we have observed when we considered poor conduting horizontal boundaries (corresponding to a smaller α), by setting $\alpha = 0.3$ in a square box with $l = 75$ and a grid of 202×202 points, corresponding to approximately 20 points per wavelength. Let us mention that the system does not settle to a single state with decrease of α and the only way to obtain results is by reducing the time increment. While the case $\alpha = 0.9$ was effectively tackled with $\tau = 0.1$, the solution for $\alpha = 0.3$ was obtained only after reducing the time increment to $\tau = 0.005$. This value of α appeaes to be a threshold, since for $\alpha = 0.2$ the numerical code is not stable even for time steps smaller than 10^{-5}. The results for the pattern evolution are shown in Fig. 3 and the L_1 norm – in Fig. 4. The pattern continuously evolves in time, as unambiguously captured by the irregular behaviour of the norm L_1, that displays a sustained high mean value.

The transition from predominantly hexagonal pattern (Fig. 1) to an irregular pattern (Fig. 3) takes place smoothly with the decrease of α. The chaotic behaviour for $\alpha = 0.4$ is qualitatively the same as for $\alpha = 0.3$. However, something like intermittency between the two regimes is observed with relatively long time spans with prédominance of the hexagonal pattern.

Finally we have studied the case of much smaller box, when the wall effects dominate the solution. Fig. 5 shows the case for a box 30×30 and resolution 52×52 for the small $\alpha = 0.4$. It is clearly seen that a rhombic pattern appears, which is not steady. Rather it is "breathing" while preserving qualitatively the main features.

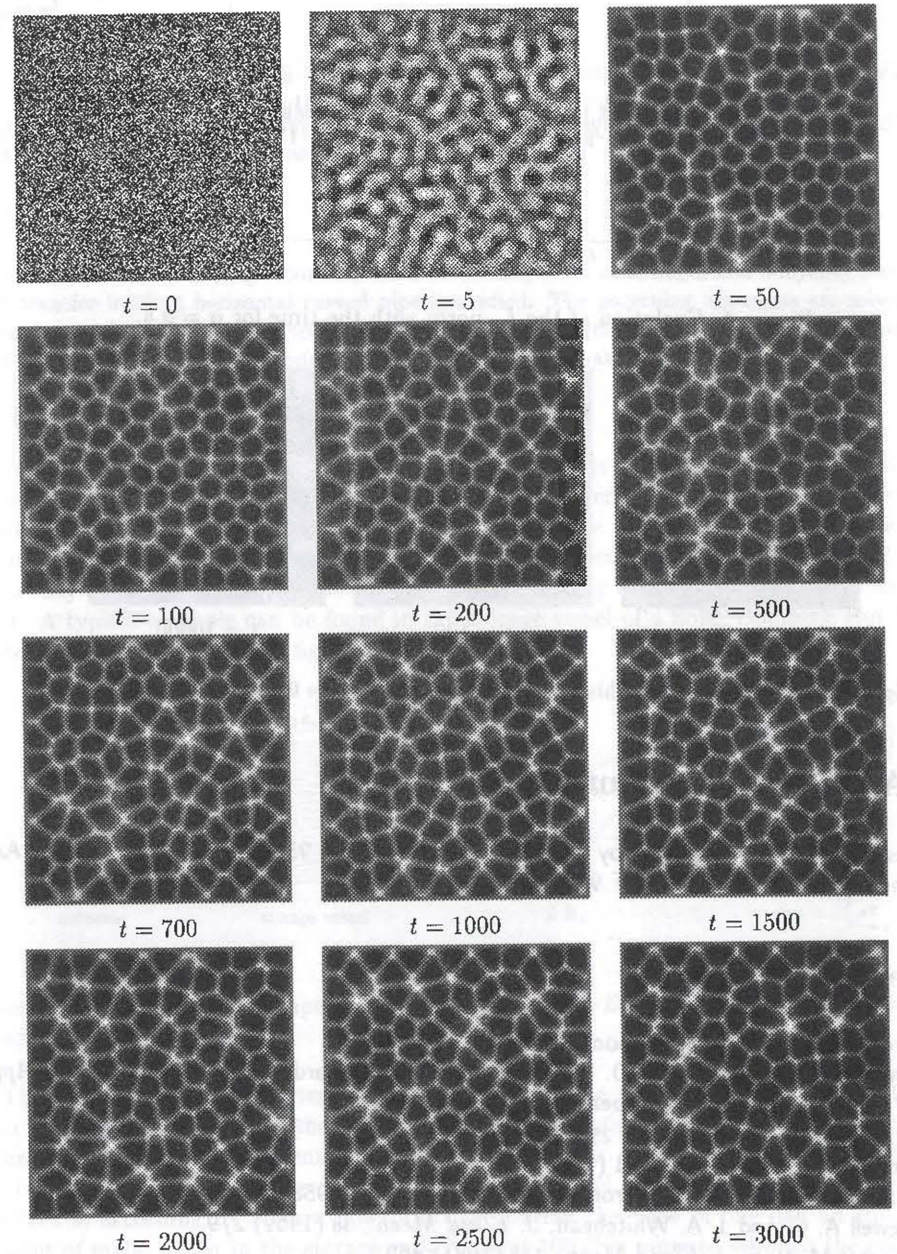


Figure 3: Evolution of the solution of eq. (1) from random initial condition with lower damping coefficient $\alpha = 0.3$.

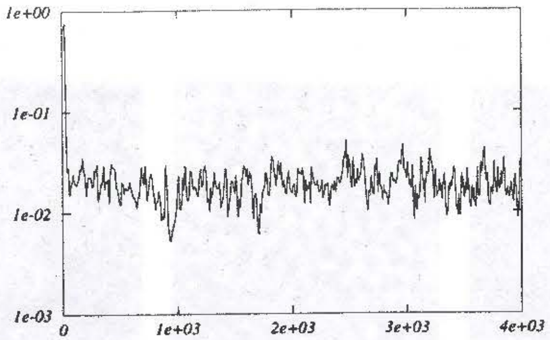
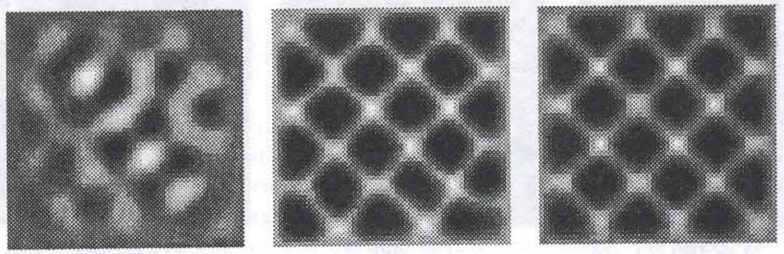


Figure 4: Evolution of the L_1 norm with the time for $\alpha = 0.3$.



$t = 20$ $t = 100$ $t = 10000$

Figure 5.The set up of "breathing" rhombic pattern for $\alpha = 0.4$ and small box 30×30.

5 Acknowledgements

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Mixed convective heat transfer in a horizontal bend

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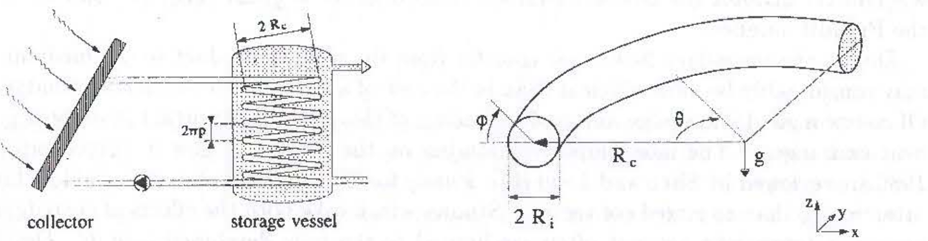
Abstract

The effect of a developing secondary flow, induced by both centrifugal and buoyancy forces, on heat transfer inside a horizontal curved pipe is studied. The governing equations are solved in a finite element formulation using triquadratic elements for the velocity as well as for the temperature field. The results show that secondary flow considerably increases heat transfer.

1. Introduction

Considering depletion of natural energy sources and its relation to pollution of the environment, it is obvious that an effective utilization of energy is of extreme importance for mankind. It is therefore not surprising that in the course of the last decennia quite some effort has been made to enhance the performance of heat exchangers. Helically coiled tubes are widely used in heat exchangers since they are relatively easy to produce, cheap and efficient. A typical example can be found in the storage vessel of a Solar Domestic Hot Water System (*SDHW*), sketched in figure 1 a.



a b

Figure 1: Coiled heat exchanger. a: sketch of a Solar Domestic Hot Water System, b: horizontally curved pipe

The collector medium (water) is pumped through the collector where it is heated by solar radiation. To tide over the time gap between collection (maximal around noon) and utilization (mostly in the evening and morning) of solar energy, heat is transfered from the collector medium to mains water in the storage vessel. An effective way to operate a *SDHWS* is according to the low-flow principle: during the time of one day an equivalent amount of mains water in the storage vessel (about 100 l) is pumped through the collector. In combination with a stratified storage, in this way exergy loss due to mixing is minimized.

In the present study we will focus on the first 90° of the coil, where the flow and temperature field develop and where the major part of the heat transfer takes place.

Since the work of Dean [5], it is known that forced laminar flow in curved ducts is characterized by secondary vortices (Dean vortices) perpendicular to the main (axial) flow. This secondary flow is a result of the centrifugal force acting on the fluid particles, which drives the